

# Modular population protocols

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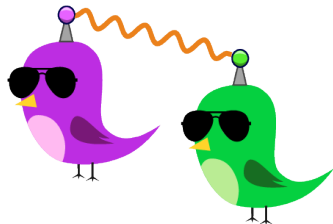
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Small sensors: constant memory, identical,  
pairwise interaction when accidentally near

Accepting and rejecting states

Aiming for eventual consensus — unanimous acceptance or rejection  
(of initial configuration)

[AADFP 2004]



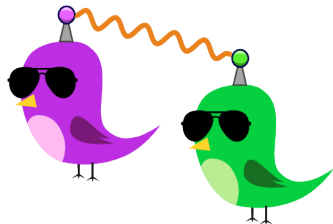
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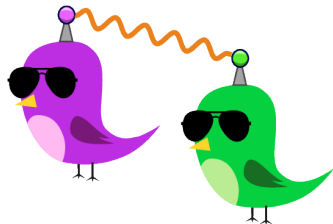
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- Generality of tasks

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## Role distribution in a distributed system

- Servers have different configurations
- Servers need to be assigned roles
- Roles have configuration requirements
- Balance of servers assigned to different roles

«Storage servers need more storage than web front-ends, but more storage on web front-ends doesn't hurt, and we need the same number of the two types whenever possible»

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- Can schedule server for shutdown and maintenance/removal
- Can add servers
- Can change server configuration

Agents have inputs they can't change, and internal states

Step options:

- Normal protocol step
- Reconfiguration

Reconfigurations:

Special input  $\perp$  means request to shutdown,

special state  $\perp$  means the agent did shut down

- Add an agent that is shut down and has shutdown input
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Two parts to specify

- Single-agent (local) input-output compatibility  
(«32 GB storage not enough for storage servers»)
- Multiset-of-inputs to multiset-of-outputs (global) compatibility  
(«Most of the servers have enough storage?  
Then webservers-to-storage-servers should be an equal split»)

After reconfigurations cease, each fair execution should eventually:

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# Expressive power

A specification can be unimplementable for specific multisets of inputs

«Exact same number of web and storage servers, even if the total is odd»

Otherwise, if global compatibility is semilinear, the specification can be implemented by population protocols with reconfiguration

A specification may be implementable for wrong reasons  
«Only web front-ends, or a prime number of storage servers»

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- Cover more of the interesting tasks...
- ... and gain modularity
- Expressive power is still «semilinear»
  
- Future work: redefine and generalize known «efficient» protocols in this form

Thanks for your attention

Questions?

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