

Exercise Sheet 1

Deformations

Our first look at topology will involve some problems about the deformation of elastic bodies and surfaces. We shall assume that the objects considered are made from a very elastic material: their shape may be changed at will, you can bend, distort, stretch, and compress them as much as you like, but of course you may not tear them or glue parts of them together.

Exercise 1.1.1. Show that the elastic body represented in Figure 1.1(a) can be deformed so as to become the one shown in Figure 1.1(b). In other words, were the human body elastic enough, after making linked rings with your index fingers and thumbs, you could move your hands apart without separating the joined fingertips.

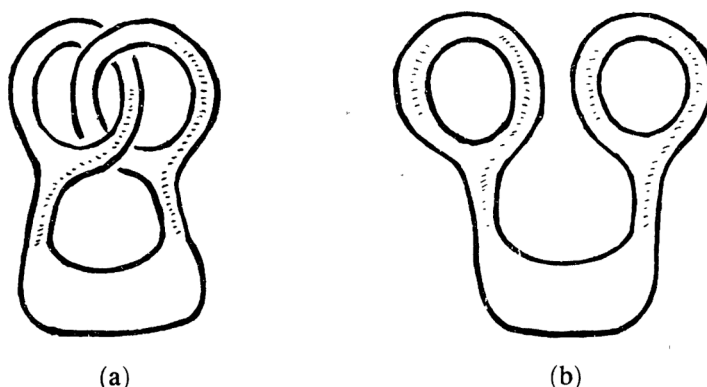


Figure 1: Linked fingers

Exercise 1.1.2. A pretzel has two holes that "hold" a doughnut (see Figure 1.2 (a)). Show that the pretzel can be deformed in such a way that one of its "handles" will unlink itself from the doughnut (Figure 1.2 (b)).

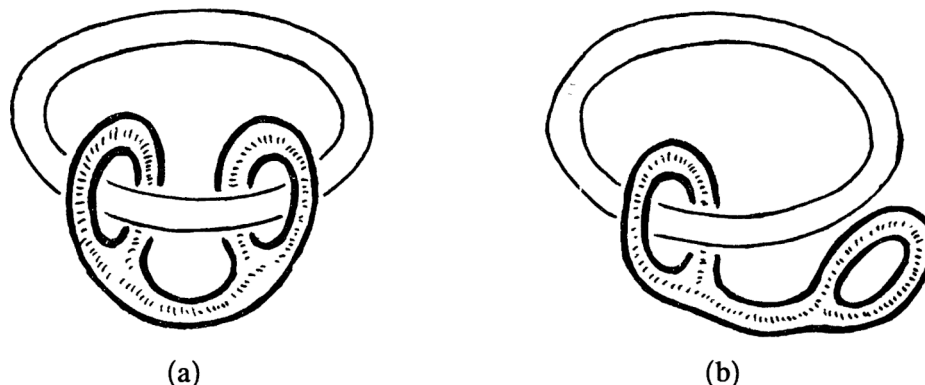


Figure 2: Pretzel and Donut

Exercise 1.1.3. Show that a punctured tube from a bicycle tire can be turned inside out. (More precisely, this would be possible if the rubber from which the tube is made were elastic enough. In real life it is impossible to turn a punctured tube inside out.)

Exercise 1.1.4. A circle is drawn on a pretzel with two holes (Figure 1.3 (a)). Show that it is possible to deform the pretzel so that the circle will be in the position represented in Figure 1.3 (b).

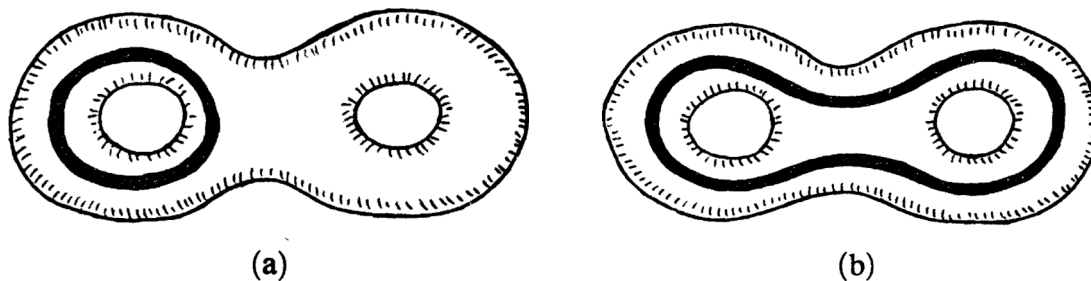


Figure 3: Pretzel with a trivial knot

Exercise 1.1.5. Show that the fancy pretzel represented in Figure 1.4 (a) can be deformed into the ordinary pretzel with two holes (Figure 1.4 (b)).

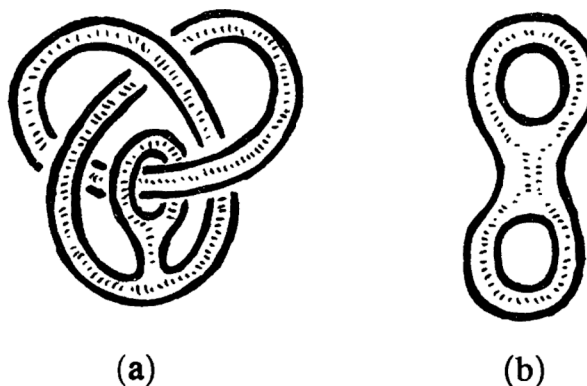


Figure 4: Fancy pretzel