LTCC: Hodge Theory

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Problem set 2-3.

You are welcome to discuss solutions with me and your classmates. If you have any questions email me at n.kurnosov@ucl.ac.uk. See you next week!

1. ow that the upper half-plane $\mathfrak{H}_g$ can be written as $Sp(g, \mathbb{Z})/U(g)$.

2. Let $g = 2$ and determine the isotropy group Γ of iId_g.

3. Show that for Kähler manifold X we have $\Delta_d = 2\Delta_{\bar{\partial}}$.

4. Consider the lattice $\Lambda_0, q_0 = \left(\mathbb{Z}^{2g}, \begin{pmatrix} 0 & -D \\ D & 0 \end{pmatrix}\right)$, where D is the diagonal matrix with elements $d_1|...|d_g$. The matrix D is called *principal polarization*. What is the space of weight one Hodge structures on Λ_0 polarized by q_0?

5. Show that the natural pairing of A and $A^\vee$ defines a morphism $A \otimes A^\vee \to \mathbb{Z}$ of Hodge structures, where $\mathbb{Z}$ is the trivial Hodge structure.

6. Check that $Sp(g, \mathbb{Z})$ preserves row space.

7. Show the Hodge diamond of a quintic in $X \subset \mathbb{P}^4$ is

			1			
		0		0		
	0		1		0	
1		101		101		1
	0		1		0	
		0		0		
			1			