LTCC: Hodge Theory

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Problem set 1.

You are welcome to discuss solutions with me and your classmates. If you have any questions email me at n.kurnosov@ucl.ac.uk. See you next week!

1. Proof that on any disk Δ in the complement of the set $\{0, 1, \infty\}$ the periods of the Legendre family are single-valued holomorphic functions.

Note: We have used it when we claimed that $P(\lambda)$ is holomorphic.

2. The function τ is non-constant.

Note: We did it in the lecture.

3. Show that τ is non-constant by showing that τ approaches infinity along the ray $\lambda > 2$ of the real axis.

Note: Show that $\tau(\lambda)$ is asymptotically proportional to log λ.

4. We proved that ω_λ and its derivative ω'_λ define linarly independent cohomology classes. Therefore thc class of the second derivative must be expressible as a linear combination of the first two classes. Let ζ be a one-cycle. Find the coefficients of the differential equation (between $\omega, \omega', \omega''$) in terms of λ.

5. Show that $\Gamma(2)$ has index six in $SL_2(\mathbb{Z})$.

6. Show that the only singular fibers of the Legendre family $y^2 = x(x-1)(x-\lambda)$ are at $\lambda = 0, 1$. Consider the family of elliptic curves $x^3 + y^3 + z^3 + \lambda xyz$. What are its singular fibers?

7. Consider the family of elliptic curves $E_{a,b,c}$ defined by $y^2 = (x-a)(x-b)(x-c)$. Waht is the locus in $\mathbb{C}^3$ of the singular fibers? Describe the monodromy representation.